



MATHEMATICAL ASSOCIATION OF AMERICA

**MAA**

## Solutions Pamphlet

American Mathematics Competitions

27<sup>th</sup> Annual

# AMC 8

American Mathematics Contest 8

Tuesday, November 15, 2011

This Solutions Pamphlet gives at least one solution for each problem on this year's exam and shows that all the problems can be solved using material normally associated with the mathematics curriculum for students in eighth grade or below. These solutions are by no means the only ones possible, nor are they necessarily superior to others the reader may devise.

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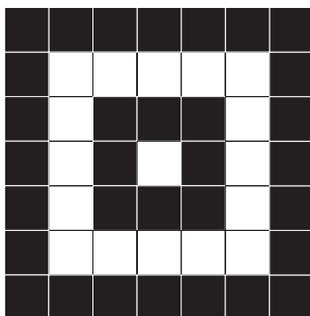
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- Answer (E):** The cost of the apples was  $3 \times \$0.50 = \$1.50$ . Her change was  $\$5.00 - \$1.50 = \$3.50$ .
- Answer (E):** Karl's garden is  $20 \times 45 = 900$  square feet. Makenna's garden is  $25 \times 40 = 1000$  square feet. Makenna's garden is larger by  $1000 - 900 = 100$  square feet.
- Answer (D):** There are 32 black tiles and 17 white tiles in the extended pattern. So the ratio is 32 : 17.



- Answer (C):** The ordered list is 0, 0, 1, 1, 2, 2, 3, 3, 3. The mean is  $\frac{15}{9} = \frac{5}{3}$ , the median is 2, and the mode is 3. Because  $\frac{5}{3} < 2 < 3$ , the correct order is mean < median < mode.
- Answer (D):** Convert 2011 minutes to 33 hours and 31 minutes. January 1 uses 24 hours. January 2 gets the remainder of the 9 hours and 31 minutes. The time at the end of 2011 minutes was 9:31AM on January 2.
- Answer (D):** In a population of 351 people, 45 people own a motorcycle. Therefore there are  $351 - 45 = 306$  car owners who do not own a motorcycle.
- Answer (C):** The upper left and the lower right squares are each one-fourth shaded, for a total of one-half square. The shaded portions of the upper right and lower left squares make up one-half square. So the total shaded area is one full square, which is 25% of the total area.

8. **Answer (B):** Make a table of possibilities.

+	1	3	5
2	3	5	7
4	5	7	9
6	7	9	11

The possible sums are 3, 5, 7, 9, and 11, for a total of 5 possibilities.

9. **Answer (E):** Carmen covers 35 miles in 7 hours, making her average speed  $\frac{35}{7} = 5$  mph.
10. **Answer (C):** Including a \$2 tip, a 0.5 mile ride would cost \$4.40. The remaining \$5.60 would take you an additional  $0.1 \times \frac{5.60}{0.2} = 2.80$  miles, so the total distance is  $0.5 + 2.8 = 3.3$  miles.
11. **Answer (A):** Asha's study time totals  $60+90+100+80+70 = 400$  minutes, for an average of  $\frac{400}{5} = 80$  minutes per day. Sasha's total is  $70+80+120+110+50 = 430$  minutes, for an average of  $\frac{430}{5} = 86$  minutes per day, so Sasha averages 6 minutes more per day than Asha.

OR

The daily differences between Sasha and Asha are +10, -10, +20, +30, and -20 minutes for a total of +30 minutes. The average difference is  $\frac{30}{5} = 6$  minutes per day.

12. **Answer (B):** Proceeding clockwise from Angie, the seating could be:  $BCD$ ,  $BDC$ ,  $CBD$ ,  $CDB$ ,  $DBC$ , or  $DCB$ . In 2 of these 6 possibilities Carlos is opposite Angie, so the probability is  $\frac{2}{6} = \frac{1}{3}$ .

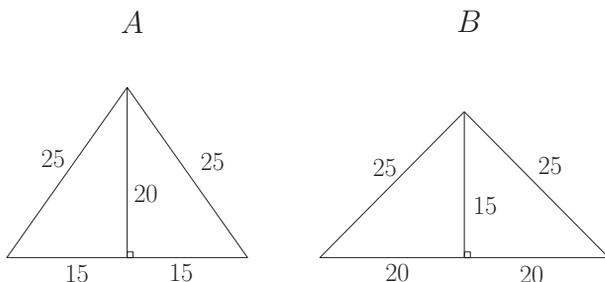
OR

If Angie sits down first, there are three equally likely places for Carlos to sit. Only one of these is opposite Angie. Thus the probability is  $\frac{1}{3}$ .

13. **Answer (C):** The shaded rectangle  $PBCS$  has height  $BC = 15$  and length  $SC = DC + SR - DR = 15 + 15 - 25 = 5$ .

Rectangle  $AQRD$  has the same height and length 25. The portion of rectangle  $AQRD$  that is shaded is  $\frac{15 \times 5}{15 \times 25} = \frac{5}{25}$ , which is 20%.

14. **Answer (C):** The number of girls at the dance is  $\frac{4}{9}(270) + \frac{5}{9}(180) = 120 + 100 = 220$ . So the fraction of the students that are girls is  $\frac{220}{450} = \frac{22}{45}$ .
15. **Answer (D):** The product  $4^5 \cdot 5^{10} = 2^{10} \cdot 5^{10} = 10^{10}$  is a number with a 1 followed by 10 zeros for a total of 11 digits.
16. **Answer (C):**



The altitude shown divides each triangle into two congruent right triangles. The hypotenuse of each right triangle is 25. In  $\triangle A$  the horizontal leg of each right triangle is 15, so the vertical leg is  $\sqrt{25^2 - 15^2} = 20$ . In  $\triangle B$  the horizontal leg of each right triangle is 20, so the vertical leg is 15. The area of  $\triangle A$  is  $\frac{1}{2}(30)(20) = 300$ , and the area of  $\triangle B$  is  $\frac{1}{2}(40)(15) = 300$ , so the two areas are equal.

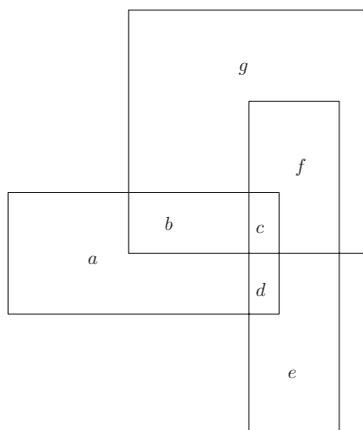
17. **Answer (A):** Factor 588 into  $2^2 \cdot 3^1 \cdot 5^0 \cdot 7^2$ . Thus  $w = 2, x = 1, y = 0$ , and  $z = 2$ , and  $2w + 3x + 5y + 7z = 21$ .
18. **Answer (D):** Make a table of 36 possible equally-likely outcomes. The first number is greater than or equal to the second in the 21 cases indicated by the asterisks, so the probability is  $\frac{21}{36} = \frac{7}{12}$ .

	1	2	3	4	5	6
1	1, 1*	1, 2	1, 3	1, 4	1, 5	1, 6
2	2, 1*	2, 2*	2, 3	2, 4	2, 5	2, 6
3	3, 1*	3, 2*	3, 3*	3, 4	3, 5	3, 6
4	4, 1*	4, 2*	4, 3*	4, 4*	4, 5	4, 6
5	5, 1*	5, 2*	5, 3*	5, 4*	5, 5*	5, 6
6	6, 1*	6, 2*	6, 3*	6, 4*	6, 5*	6, 6*

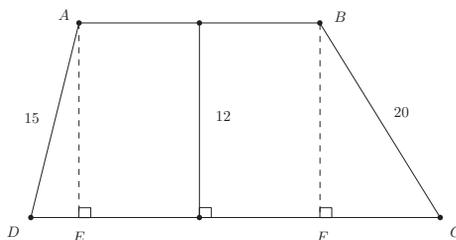
OR

In 6 of the 36 possible outcomes the two numbers are equal. The first number is greater than the second in half of the remaining 30 outcomes, so the first number is greater than or equal to the second in  $6 + 15 = 21$  outcomes. The probability is  $\frac{21}{36} = \frac{7}{12}$ .

19. **Answer (D):** Partition the figure into non-overlapping regions as shown. The list of rectangles are  $b$ ,  $c$ ,  $d$ ,  $ab$ ,  $bc$ ,  $cd$ ,  $cf$ ,  $de$ ,  $abcd$ ,  $bcfg$ , and  $cdef$ .



20. **Answer (D):** Let  $E$  and  $F$  be the feet of the perpendiculars from  $A$  and  $B$  to  $\overline{DC}$ . In right  $\triangle AED$ ,  $DE^2 = 15^2 - 12^2 = 225 - 144 = 81$ , so  $DE = 9$ . In right  $\triangle BFC$ ,  $FC^2 = 20^2 - 12^2 = 400 - 144 = 256$ , so  $FC = 16$ .



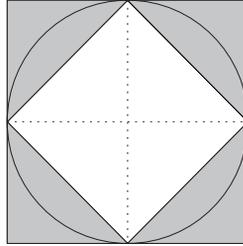
Right  $\triangle AED$  has area  $\frac{1}{2} \cdot 9 \cdot 12 = 54$ , right  $\triangle BFC$  has area  $\frac{1}{2} \cdot 16 \cdot 12 = 96$ , and rectangle  $ABFE$  has area  $50 \cdot 12 = 600$ . The trapezoid  $ABCD$  has area  $54 + 96 + 600 = 750$ .

OR

Begin as in the first solution and note that  $DC = DE + EF + FC = 9 + 50 + 16 = 75$ . Then the area of trapezoid is  $\frac{1}{2}(AB + DC) \cdot AE = \frac{1}{2}(50 + 75) \cdot 12 = 125 \cdot 6 = 750$ .

21. **Answer (C):** Because half are too low, Norb is at least 37. Because two are off by one, he must be between 36 and 38 or between 47 and 49. Because 37 is prime and 48 is not, Norb is 37.
22. **Answer (D):** The tens digit of a power of 7 is determined by the last two digits of the previous power of 7. The pattern for the last two digits of successive powers of 7 is **01, 07, 49, 43, 01, 07, 49, 43, 01, 07, 49, 43, 01, ...** Since  $2011 = 4 \cdot 502 + 3$ , the last two digits of  $7^{2011}$  are 43 and the tens digit is 4.
23. **Answer (D):** Any integer that is a multiple of 5 must have a 0 or 5 as the units digit.
- If the units digit is 0, then the other three digits must be a 5 and two digits selected from 1, 2, 3, and 4. After the pair is selected, there are 6 possible ways to arrange them to form the number. So there are  $6 \cdot 6 = 36$  possible numbers with units digit 0.
- If the units digit is 5, then there are four choices (1, 2, 3, 4) for the thousands digit and there are  $4 \cdot 3 = 12$  ways to complete the number. So there are  $4 \cdot 12 = 48$  possible numbers with units digit 5.
- Together there are  $36 + 48 = 84$  possible numbers.
24. **Answer (A):** If the sum of two numbers is odd, one number must be even and the other number must be odd. Because all primes except 2 are odd, 2 must be one of the summands. Because  $10,001 = 2 + 9999$ , and  $9999 = 9 \cdot 1111$  is not prime, there are no solutions.

25. **Answer (A):** The area of a circle of radius 1 is  $\pi(1)^2 = \pi$ . The side length of the big square is the diameter of the circle, which is 2, so its area is  $2^2 = 4$ . The big square can be divided into 8 congruent triangles, and the shaded area is made up of 4 of those triangles. The shaded area is half the area of the big square, which is 2. The requested ratio of the two shaded areas is  $\frac{\pi-2}{2} \approx \frac{3.14-2}{2} \approx \frac{1}{2}$ .



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